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Entropy Studies of Photons in Pb-Pb Interactions at 158 A GeV

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Abstract—*The motivation for studying relativistic heavy ion collisions is to gain understanding of equation of state of nuclear, hadronic and partonic matter commonly referred to as nuclear matter. Such a situation is quite suitable for the formation of a deconfined state of matter known as hot Quark-Gluon Plasma (QGP) which subsequently cools and expands. In this process the energy density becomes low enough so as a phase transition Quark-Gluon Plasma (QGP) to hadrons state occurs. Entropy produced in this dynamical system is analyzed at SPS energies and its dependence on the number of bins and on the size of phase space region is investigated.*

Keywords: *Quark-Gluon Plasma (QGP), Heavy ions, Equation of state.*

1. INTRODUCTION

Entropy, being one of the most important characteristics of a system with many degrees of freedom, is in particular - an important parameter of multiparticle production processes. In this context it abounds in analysis of dense hadronic matter and in discussions of various models of Quark Gluon Plasma [1]. Entropy gets produced in the so-called dynamical systems [2,3] envisaging multiparticle creation. Although application of the mathematical theory of dynamical system to calculate the entropy in multiparticle production is still out of reach, the existing models suggest that the systems produced in high energy collisions pass through a stage of (approximate) local statistical equilibrium [4].

In ultra-relativistic heavy ion interactions, entropy analysis is reported in literature with regard to information entropy, Renyi information entropies, conditional and mutual information entropies [5-7]. New theoretical proposals are reported to study entropy variable by making use of coincidence method [8-10]. Basic idea behind this proposal is to study event-by-event fluctuations by the determination of entropy of multiparticle systems created in High Energy Collisions. The proposal should be important for the analysis of RHIC and LHC experiments where it may help in the separation of a possible signals from Quark-Gluon-Plasma (QGP) In the present analysis we use the Photon Multiplicity Detector (PMD) data (79005 central events) from WA98 experiment at CERN SPS, random data events and mixed data events to create samples of multiparticle states and analyze them according to the proposal mentioned above. We explain the procedure and variable used in the analysis. The experimental setup and results are of the results including some conclusions and perspectives. Quark Gluon Plasma. The condition to achieve such a new state is to produce a sufficiently hot and dense bulk of nuclear matter. Relativistic heavy Ion Collider at BNL and Large Hadron Collider (LHC) at CERN.

2. MATHEMATICAL FORMULATION

As the first step in the process of measurement, one has to select a phase space region in which measurements are to be performed. This of course depends on the detector acceptance as well as on the physics one wants to investigate. The selected phase space region is divided into bins of equal size in pseudo-rapidity space(η). The number of bins cannot be too large if one wants to keep errors under control. On the other hand, it is important to study the dependence of results on the size (and thus the number) of bins. Therfore, large statistics is essential for a meaningful analysis. Using this procedure, an event is characterized by the number of particles in each bin i.e by a set of integer numbers s= $m_i^{(j)}$, where i=1,2,….M(M is the total number of bins) and the superscript (j) runs over all kinds of particles present in the final state. These sets represent different states of the multiparticle system. To calculate the entropy we discretize each data events using bins in rapidity space. number of particles in each bin m_i , i=1,2,…….M is recorded. Shann0on entropy is calculated from the standard def

$$S= - \sum p_j \log p_j$$

$$p_j = \frac{n_j}{N}$$

Where p_j denotes the probability to obtain any specific configuration of numbers $\{m_i\}$

Where n_j are the number of events providing such configurations and N is the global number of events. The measurement consists of simple counting how many times (n_j) any given set j appears in the whole sample of events. Once numbers n_j are known for all sets, one calculates total number of observed coincidencesof k configurations

$$N_k = \sum n_j(n_j-1)\ldots\ldots\ldots..(n_j-k+1)$$

Where k=1,2,3,…….The sum formally runs over all sets recorded in a given experiment, but non vanishing contributions given only for those which were recorded atleast k times. N_k is the total number of observed coincidences of k configurations. The coincidence probability of k configuration is

$$\sum n_j = 1,\ C_1 =1$$

Finally, One can see that only states with $n_j \geq k$ contribute to N_k (and thus also to C_k)

Once the coincidence probabilities C_k (k=1,2,3,….) are measured, it isconvenient to calculate

Renyi entropies by[11]

$$H_k = -\frac{\log C_k}{k-1}$$

One can also calculate Renyi entropies directly from the following definition

$$H_k = -\sum_j (p_j)^k$$

The Shannon entropy is formally equal to the limit of Renyi entropies H_k as k⟶1 and can be obtained by extrapolation. Obviously N_1 = N, C_1 = 1 and this extrapolation cannot be done just by putting k=1 in formula given by equation. It is suggested in [10] to use for the extrapolation formula

$$H_k = a \log k/ (k-1) + a_0 + a_1(k-1) + a_2(k-1)^2 +$$

Where the number of terms is determined by the number of measured Renyi entropies. Usually it is enough to use H_k for k=2,3,4. Other extrapolation can also be used e.g

$$H_k = a_0 + a_1k^{-1} + a_2k^{-2} + \ldots\ldots.$$

For the free gas of massless bosons,Renyi entropies and Shannon entropies are related by

$$H_k = (1+\frac{1}{k}+\frac{1}{k^2}+\frac{1}{k^3})\frac{S}{4}$$

Where S is the Shannon entropy and should be compared with one presented here to estimate the extrapolation accuracy. These entropies will depend on the method of discretization of η space, in particular on the size of binning. If the bins are small enough and if the system is close to thermal equilibrium one expects the following scaling laws to hold

$$H_k(lM) = H_k(M) + \log l\ ;\ S(lM) = S(M) + \log l$$

This scaling rule is not expected to hold if the system is far from the thermal equilibrium and the fluctuations of the particle distributions are large. In particular, the effect of intermittency [12] and erraticity [13] as implied, e.g., by a cascading mechanism of particle production are expected to violate this Eq.. Thus testing the dependence of entropies on the number of bins may reveal interesting features of the system. Another expected feature is additivity: for entropies measured in a phase space region R,which is the sum of two regions R_1 and R_2, we should observe

$$H_k(R) = H_k(R_1) + H_k(R_2)\ ;\ S(R) = S(R_1) + S(R_2)$$

The additivity features should be satisfied if there are no strong correlations between the particles belonging to the regions R_1 and R_2. Thus the verification of additiveness gives information about the correlations between different phase space regions.In short the simplest tests of the general scaling and additive rules can provide essential information on fluctuations and correlations in the system. We check these features by choosing different number of bins in different ranges of pseudo-rapidity (η)

3. WA98 EXPERIMENT

Photon Multiplicity Detector [14] is situated at 21.5 meters from the target and covers a pseudorapidity range $2.8 < \eta < 4.4$ .This consists of a Lead (Pb) converter sheet of thickness equal to three radition length with iron frame supports in front of an array of ~ 54000 scintillation pads of square geometry of varying sizes (15mm, 20mm and 25mm) and of thickness 3mm arranged in 22 light tight boxes around the beam axis. Each box contains a matrix of 50x38 pads. Photons falling on the PMD materialize in the lead converter and produce partially grown electromagnetic showers which fall on the plastic scintillator pads and produce light, the light so produced is transported through wave length shifting (WLS) fibres which were subsequently thermally bonded th clear fibres. The WLS fibre from each box are bunched together through holey plate and the read out is done by image intensifier (II) and CCD camera. A threshold of 3 MIP on the preshower ADC content is used to correct for background due to charged particles and number of gamma-like clusters is estimated. This gives an average photon counting efficiency of 70-75% depending on the centrality which is almost uniform over the eta –range under consideration. Statistical fluctuations are on the top of the dynamical fluctuations in any real experiment so the distribution of entropy becomes wider and average entropy smaller. This effect is weaker for higher multiplicities. At sufficiently high multiplicity, we mar expect the influence of statistical fluctuations will become negligible. In order to estimate the contributions from statistical fluctuations, we generated a sample of correlation free ensemble of events i.e "Mixed Events" and correlation free MONTE CARLO events (MC-RAND) Mixed event technique is a very powerful technique as it allows one to generate totally correlation free ensemble of events. The sample out of which the mixed events are to be generated is read and stored in the program. The events and tracks are randomly picked up to generate mixed events matching the number and multiplicity structure of the experimental data. The restriction on multiplicity will constrain the mean and sigma of N_{gamma} – distributions of mixed event to exactly like that of N_{gamma}- distributions of real data. This restriction is used to avoid any biases, which can crop in. The other restriction put in generating the mixed event is that, no two particles in an event should have : ($\Delta\eta \leq 0.01$ and $\Delta\Phi \leq 1^{0}$). The mixed events thus generated are subjected to the calculations.

Correlation free MC events (MC-RAND) are generated by applying the criteria

(a) The multiplicity distribution of the emitted particles should be similar to those obtained for the experimental data.

(b) There should be no correlation between the particles produced and

(c) For each event the single particle inclusive distribution in η space is set to have Gaussian shape with its mean value and dispersion comparable to the corresponding experimental values obtained for the entire sample [15]

4. DETAILS OF CALCULATIONS

The experimental data from heavy-ion hybrid CERN SPS based The values of Shannon entropy from the direct definition and Renyi's entropies corresponding to configurations k=2(H_2) and k=3(H_3) are calculated for a number of bins M varying between 2 and 5. A plot is shown in the Fig. 1. A progressive increase in values with increase in the number of bins is observed.The increase however saturates for 4≤ M ≤ 5. For a system close to equilibrium, entropy values are expected to grow logarithmically with the number of bins (along ln M). The dependence of the entropy on the number of bins seems to be stronger than the predicted by equation i.e in an expected logarithmic increase which is an indication of absence of thermal equilibrium in the process.

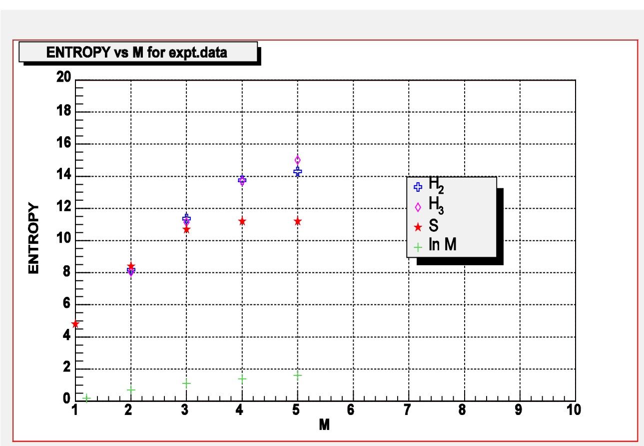

Fig. 1: Entropy variations with no. of bins for expt.data

Since it was suggested that additivity may be broken by correlation effects, we checked if short-range correlations are relevant .For this, we calculated the entropies for the same number of bins in Φ using the rapidity range $\Delta\eta$=0.2 in "one piece"(3.4≤η≤3.6) and in two intervals of width 0.1 separated by gap of 0.2 units(3.3≤η≤3.4 and 3.6≤η≤3.7). The results are shown in the Fig. 2 for experimental data which are barely distinguishable for adjacent and split region, indicating the absence of short range correlations. ln M is plotted in the Fig. for comparison. Fig. 3 and 4 shows the plots for Random data and Mixed data events for adjacent and split regions, we find almost the reproduction of the experimental data events indicating no fluctuations from non-statistical sources.

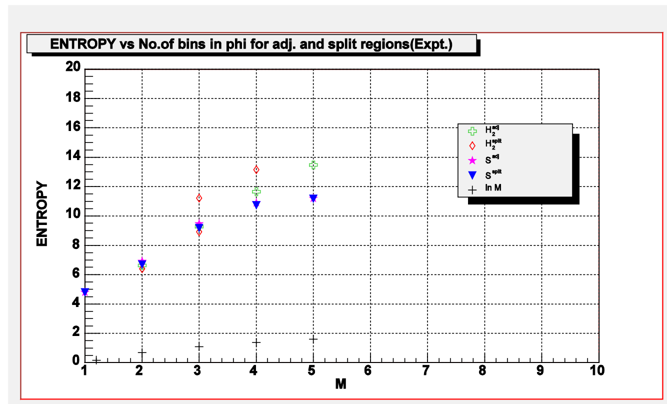

Fig. 2: Entropy variations with no. of bins for adjacent and split regions for expt. Data

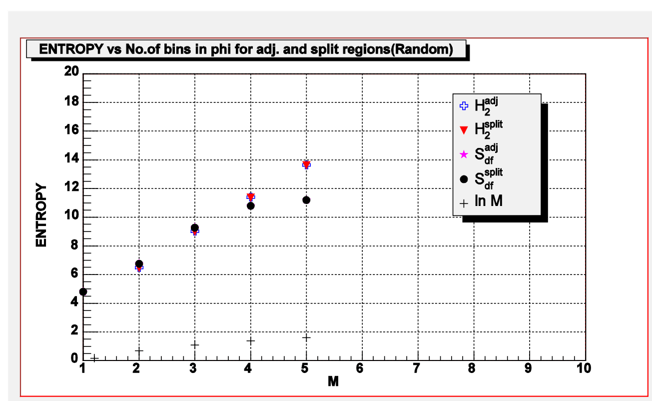

Fig. 3: Entropy variations with no. of bins for adjacent and split regions for random Data

Graphics to the data are obtained using standard ROOT CERN software. The statistical errors are observed to be significantly small.

In summary, the dependence of entropy on the number of bins is observed. It seems to be stronger than the logarithmic growth expected for thermal equilibrium scenario. Comparison with statistical data i.e Random and Mixed data events indicates qualitative and quantitative agreement. The conjecture that entropy is additive is tested i.e that entropy measured in a phase space region R which is the sum of two regions R_1 and R_2 is just the sum of entropies measured in these two regions . Our results confirm this conjecture.

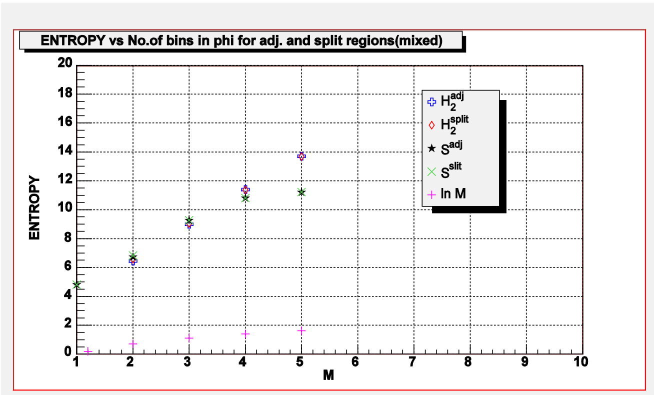

Fig. 4: Entropy variations with no. of bins for adjacent and split regions for mixed Data

5. SUMMARY AND CONCLUSIONS

The characteristics observed with experimental results in extensive variable entropy of "fireball" in ultra-relativistic interactions of Pb-Pb at 158 A GeV are the manifestations of algorithm, known collision physics and statistics. We find almost the reproduction of the experimental data results indicating no fluctuations in pseudo-rapidity distributions of photons which could be attributed from non-statistical sources. The variations found were well within errors and could be appreciated only in low multiplicity occupancy regions.

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